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PROBLEMS AND SOLUTIONS.

SEND ALL COMMUNICATIONS ABOUT PROBLEMS TO B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

ALGEBRA.

487. Proposed by WILLIAM HOOVER, Columbus, Ohio.

Show in two ways that 0.5623 is not a root of $(1 - m)e^m = 2e^{-1}$, e being the Napierian base. Find the correct value of m, and test the result in two ways.

488. Proposed by NORMAN ANNING, Chilliwack, B. C.

Show that

$$\begin{vmatrix} a_1 & 1 & a_2 \\ 1 & 1/a_4 & 1 \\ a_2 & 1 & a_3 \end{vmatrix} = 0,$$

where

$$a_k = \frac{\sin(k\theta + \alpha)}{\sin k\theta},$$

and θ and α have any values that do not make a denominator zero.

GEOMETRY.

520. Proposed by ALBERT A. BENNETT, University of Texas.

On a given tangent to a circle determine a point such that, if a secant be drawn joining this point to the extremity of the diameter which is perpendicular to the given tangent, the segment of this secant exterior to the circle will be equal in length to a given segment.

521. Proposed by R. M. MATHEWS, Riverside, California.

A variable circle, with center on the line l and passing through a fixed point P, cuts a fixed circle in A and B. Prove that the common chord AB and the perpendicular to l through P intersect in a fixed point.

CALCULUS.

435. Proposed by B. F. FINKEL, Drury College.

Show that

$$\int_0^\infty e^{-x^2-(a^2/x^2)}dx = \frac{\sqrt{\pi}}{2e^{2a}}$$

by a transformation, rather than by the usual method of differentiating under the sign of integration, as, for example, in Byerly's *Integral Calculus*, pages 106–107.

436. Proposed by ARTEMAS MARTIN, LL.D., Washington, D. C.

A circle of radius a is drawn at random on a circular slate of radius r. If another circle of radius a be drawn on the slate, what is the probability that the second circle will intersect the first?

MECHANICS.

352. Proposed by C. N. SCHMALL, New York City.

A glass rod is balanced partly in and partly out of a cylindrical tumbler, with its lower end resting against the vertical wall of the tumbler. If ϕ and ψ are the maximum and minimum angles, respectively, which the rod can make with the vertical plane, and θ is the angle of friction, show that

$$\theta = \tfrac{1}{2}\tan^{-1}\left(\frac{\sin^3\phi - \sin^3\psi}{\sin^2\phi\cos\phi + \sin^2\psi\cos\psi}\right).$$

353. Proposed by CLIFFORD N. MILLS, Brookings, North Dakota.

A uniform beam of oak, 10 feet long, 15 inches deep and 10 inches wide, sustains, in addition to its own weight, a load of 5,000 lbs. placed at the center. Find the greatest bending moment and the greatest stress in the fibers. Take the specific gravity of oak as .934.

NUMBER THEORY.

270. Proposed by GERSHOM N. CARMICHAEL, Urbana, Ill.

Does there exist a fraction p/q in its lowest terms such that the ratio of the sum of the divisors of p to the sum of the divisors of q is equal to p/q? Give a method of finding such fractions not in their lowest terms.

271. Proposed by HORACE OLSON, Chicago, Ill.

Prove that if x, y, z, u, v, and w are integers such that $x^2 + y^2 = u^2, x^2 + z^2 = v^2, y^2 + z^2 = w^2$, then the product $xyzuvw$ is divisible by 518400.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

474. Proposed by A. A. BENNETT, University of Texas.

Show that the value of the infinite continued fraction, all of whose coefficients are unity,

$$1 + \frac{1}{1+}\,\frac{1}{1+}\cdots, \quad \text{is} \quad \tfrac{1}{2}(1+\sqrt{5}).$$

Also find an explicit algebraic formula for the nth convergent.

SOLUTION BY C. C. YEN, Tangshan, North China.

1. Let x denote the value of the continued fraction. Then,

$$x = 1 + 1/x, \quad \text{i. e.,} \quad x^2 - x - 1 = 0,$$

therefore,

$$x = \tfrac{1}{2}(1 + \sqrt{5}),$$

where the sign for the radical is positive, since x is evidently positive.

2. Let p_n/q_n denote the nth convergent. Then

$$p_1 = 1, \qquad q_1 = 1, \qquad p_2 = 2, \qquad q_2 = 1. \tag{I}$$

Also, since all the coefficients are unity,

$$p_n = p_{n-1} + p_{n-2}, \qquad q_n = q_{n-1} + q_{n-2} \qquad (n = 3, 4, 5, \cdots). \tag{II}$$

And, it follows, therefore, from I and II,

$$q_n = p_{n-1} \qquad (n = 2, 3, 4, \cdots). \tag{III}$$

Now consider the series

$$1 + x + 2x^2 + p_3x^3 + p_4x^4 + \cdots + p_nx^n + \cdots,$$

where the coefficients p_n satisfy II. It is a recurring series whose scale of relation is $1 - x - x^2$, and whose generating function is $1/(1 - x - x^2)$. Hence, p_n is the coefficient of x^n of the expansion of this function; and, by III, q_n is the coefficient of x^{n-1} of the same expansion.

If we put $\alpha = -\frac{1}{2}(1 + \sqrt{5}), \beta = -\frac{1}{2}(1 - \sqrt{5})$, we get

$$\frac{1}{1 - x - x^2} = \frac{1}{\alpha - \beta}\left(\frac{1}{\alpha - x} - \frac{1}{\beta - x}\right),$$

whence the general term of the expansion of $1/(1 - x - x^2)$ is

$$\frac{1}{\alpha - \beta}\left\{\left(\frac{1}{\alpha}\right)^{n+1} - \left(\frac{1}{\beta}\right)^{n+1}\right\}x^n = \frac{1}{\alpha - \beta}\left\{\frac{\beta^{n+1} - \alpha^{n+1}}{(\alpha\beta)^{n+1}}\right\}x^n.$$